

Clex 3: Discrete Random Variables

Problem 1: Sensor reliability

A network administrator monitors the number of connectivity drops X experienced by an IoT gateway over a 24-hour period. The probability distribution is as follows:

x	0	1	2	3	4	5
$P(X = x)$	0.45	0.25	0.15	0.08	0.05	0.02

1. Find the expected number of drops $E[X]$.
2. Calculate the variance $\text{Var}(X)$.

Problem 2: Optical lenses

A manufacturing process for optical lenses has a defect rate of $p = 0.0015$. A batch of $n = 2000$ lenses is produced.

1. Identify the parameters for the Binomial distribution modeling the number of defects X .
2. Define the parameter λ for a Poisson approximation.
3. Approximate $P(X = 3)$ using the Poisson formula: $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$.

Problem 3: API requests

Context: A cloud service provider classifies incoming API requests into four distinct types:

- C_1 : Read requests ($p_1 = 0.60$)
- C_2 : Write requests ($p_2 = 0.20$)
- C_3 : Delete requests ($p_3 = 0.14$)
- C_4 : Admin requests ($p_4 = 0.06$)

Let $\mathbf{X} = (X_1, X_2, X_3, X_4) \sim \text{Multinomial}(n = 27, \{p_1, p_2, p_3, p_4\})$.

1. **Marginal Distribution:** Prove that the marginal distribution of X_1 is binomial. What are the parameters of this binomial distribution?
2. **Conditional Distribution:** Prove that the conditional probability mass function $P(X_2, X_3, X_4 \mid X_1 = 18)$ corresponds to a multinomial distribution. What are the parameters of this multinomial distribution?
3. Compute $P(X_2 = 5, X_3 = 3, X_4 = 1 \mid X_1 = 18)$