Clex 4: Continuous Random Variables

Problem 1: Corridor Aérobique

Morin-Heights is the starting point for Corridor Aérobique which during the winter months is a $58km$ long cross-country skiing trail. Yvan & Co. ski on this trail frequently. The distance covered during these excursions is well-modelled by a random variable with a probability density function

$$f(x) = \begin{cases} (x-12)^2/1152 & 12 \leq x < 24 \\ 9/(x-12)^2 & 24 \leq x \leq 48 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the cumulative distribution function of the distance covered.
b) Sketch the graph of the cumulative distribution function.
c) Find the ninety-eighth percentile, P_{98}, of the distance covered.
d) Determine the average distance covered.

Problem 2: Rates of calls and exponential waiting

A physicians office receives an average of six calls per hour.

1. What is the probability that the office will receive no calls in the next 10 minutes?

2. What is the probability that there will be at least one phone call within the next half hour?

3. If no phones calls were received at the office in the last 20 minutes, what is the probability that one will be received in the next 8 minutes?

4. How many minutes would the office have to wait in order to get at least one phone call with 99% probability?

Problem 3: Standardization

Show that if X is a normal random variable with mean μ and st.dev σ then $Z = (x-\mu)/\sigma$ is standard normal.
Take the following steps:

1. Write the cdf $F_Z(z)$ in terms of the cdf of the random variable X.

2. Change appropriately the variable of integration in the resulting integral expression and observe that the integral defines the cdf of the standard normal.