

## Clex 7: Directional derivatives

### Problem 1: Directional derivative

In what directions at the point  $(2, 1)$  does the function  $f(x, y) = xy$  have a rate of change  $-1$ ? Are there directions in which the rate is  $-3$ ?

### Problem 2: Ant cooling

The temperature  $T(x, y)$  at points of the  $xy$ -plane is given by  $T(x, y) = x^2 - 2y^2$ .

- Draw the isotherms (level curves for  $T$ ).
- In what direction should an ant in position  $(2, -1)$  move if it wishes to cool off as quickly as possible?
- Along what curve though  $(2, -1)$  should the ant move in order to continue to experience maximum rate of cooling?

### Problem 3: Fly seeking heat

The temperature in 3d space is given by

$$T(x, y, z) = 24 - x^2 - y^2 - z^2 + 2xz + 2yz$$

The fly is hovering at point  $(1, 0, 1)$ . Unexcusably the fly has forgotten how to integrate so finding the best path to the max heat analytically is not an option. Thankfully the fly computes partial derivatives 'on the fly'. Swallowing her pride the fly decides to hop along the current gradient. The magnitude of the hops is equal to the magnitude of the gradient.

- What was the temperature initially?
- Where is the fly after one hop? What is the temperature at this point?
- Where is the fly after two hops? What is the temperature at this point?
- Where is the fly after three hops? What is the temperature at this point?