Clex 8: Differentials and Jacobians

Problem 1: Linear approximation via a Jacobian

Determine the Jacobian matrix $D\mathbf{f}(x, y, z)$ for the function $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$f(x, y, z) = (x^2 + yz^2, y - x \ln z)$$

Use $D\mathbf{f}(2, 2, 1)$ to approximate the value for $\mathbf{f}(1.95, 2.1, 0.98)$,

Problem 2: Composition of Jacobians

Consider the function $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^2$ given by $\mathbf{f}(x, y, z) = (y/x, z/y)$ and the function $\mathbf{g} : \mathbb{R}^2 \to \mathbb{R}^3$ given by $\mathbf{g}(u, v) = (1 + u/v, 1 - v/u, u + v)$.

- Determine the Jacobian of $\mathbf{f}$ at the point $(1, 2, 1)$.

- Determine the Jacobian of $\mathbf{g}$ at the point $(2, \frac{1}{2})$.

- Determine the Jacobian of $\mathbf{g} \circ \mathbf{f}$ at the point $(1, 2, 1)$.

- Illustrate the validity of the chain rule formula on this example.